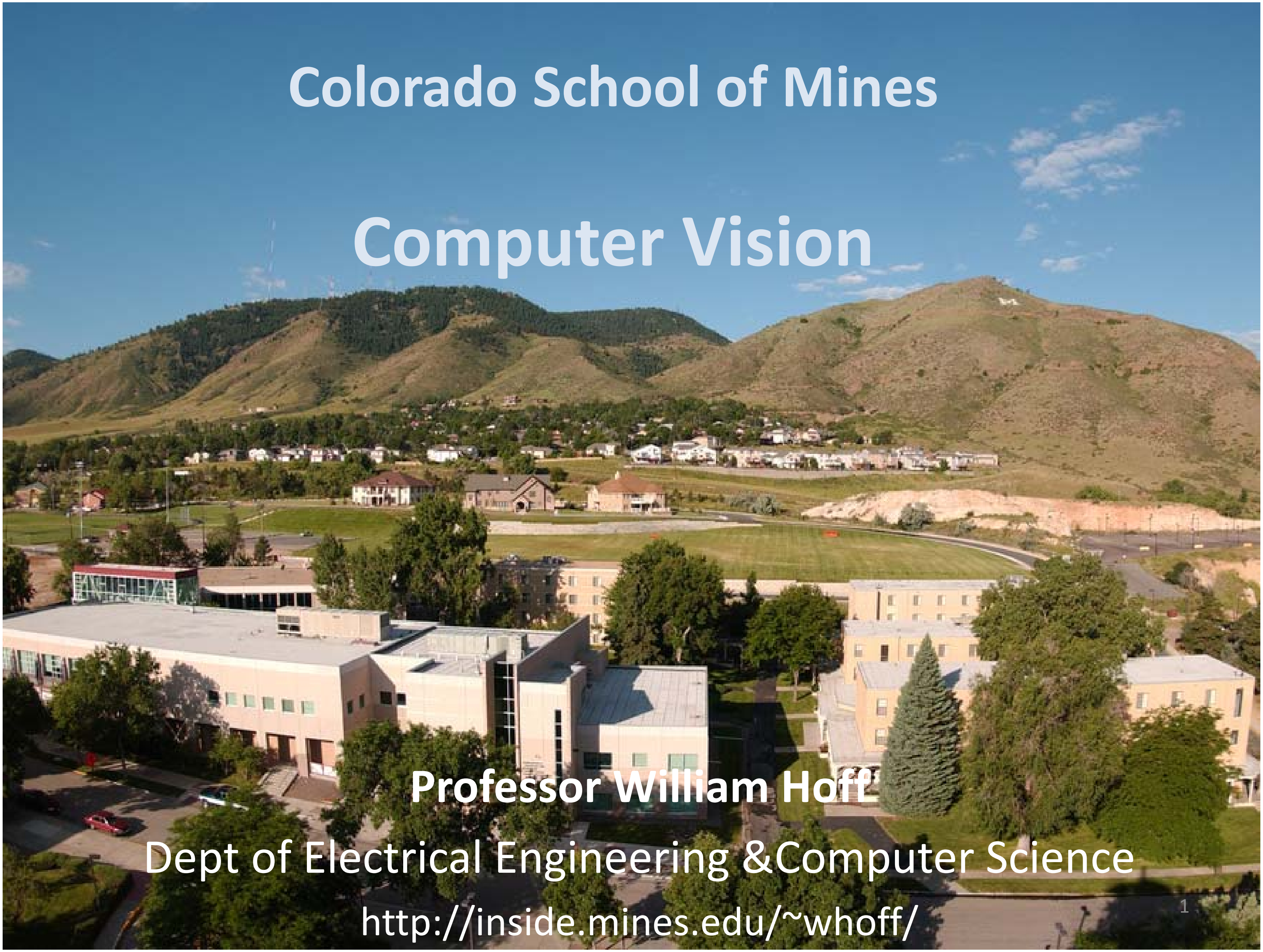

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Finding a Checkerboard

Checkerboard Calibration Patterns

- Most approaches in the literature are focused on finding checkerboards used for camera calibration
- These are very clean images, and a variety of methods work well
- The methods may not work well for actual images of checkers and chess games

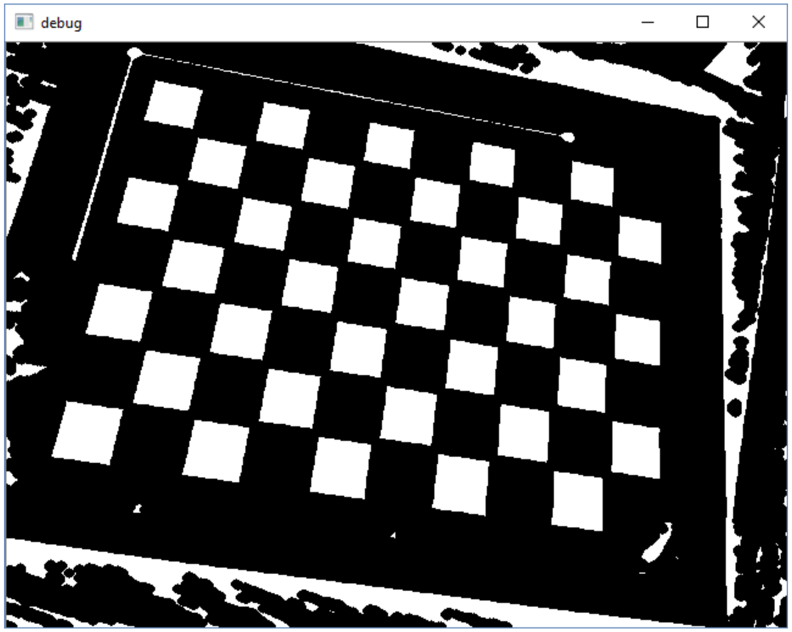
Real game images

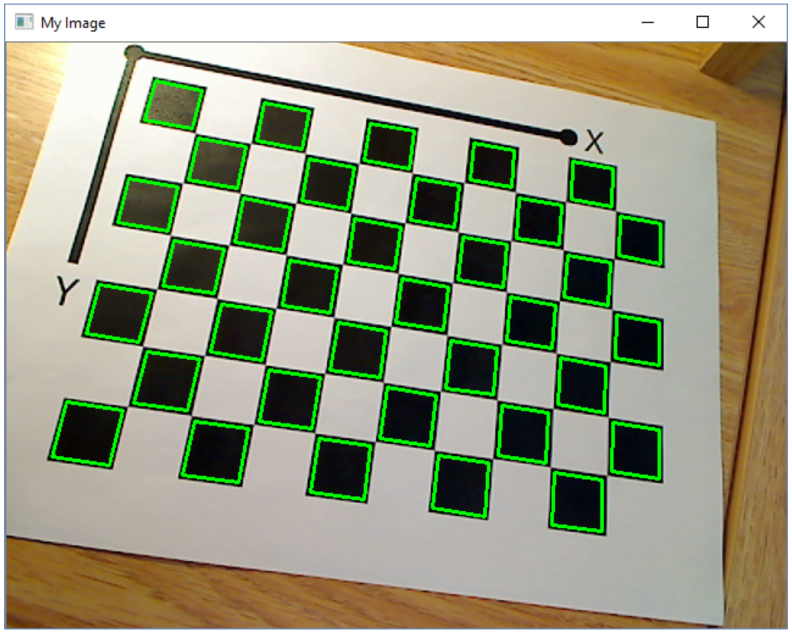
Finding Checker/Chess Gameboards

- Potential problems
  - Board may be partially occluded or out of field of view
  - Lighting problems: glare, shadows
  - There may be pieces partially covering the squares
- Helpful facts
  - Lines are prominent (long lines, high contrast edge points)
  - Geometry is known (9x9 lines, pattern of b/w squares)
- Assumptions
  - Board is almost all visible (there isn't much occlusion)

Approach: Find Squares

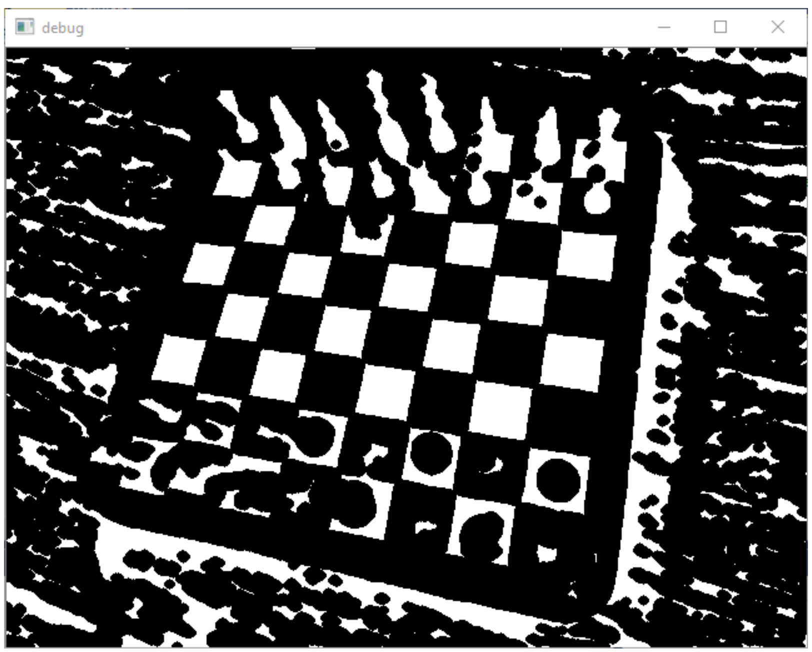
- Threshold image
- Find contours around white regions; approximate with line segments; keep those with 4 sides

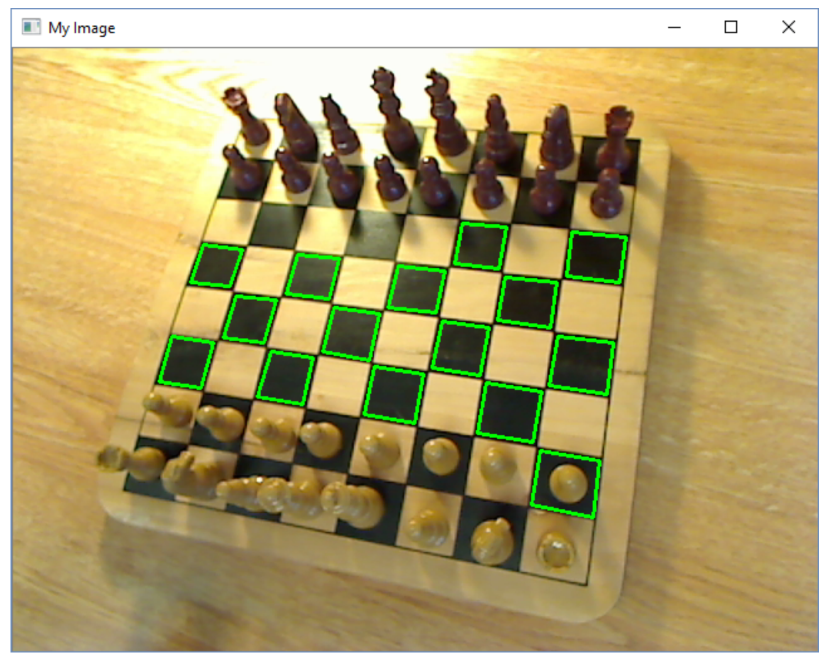




Approach: Find Squares

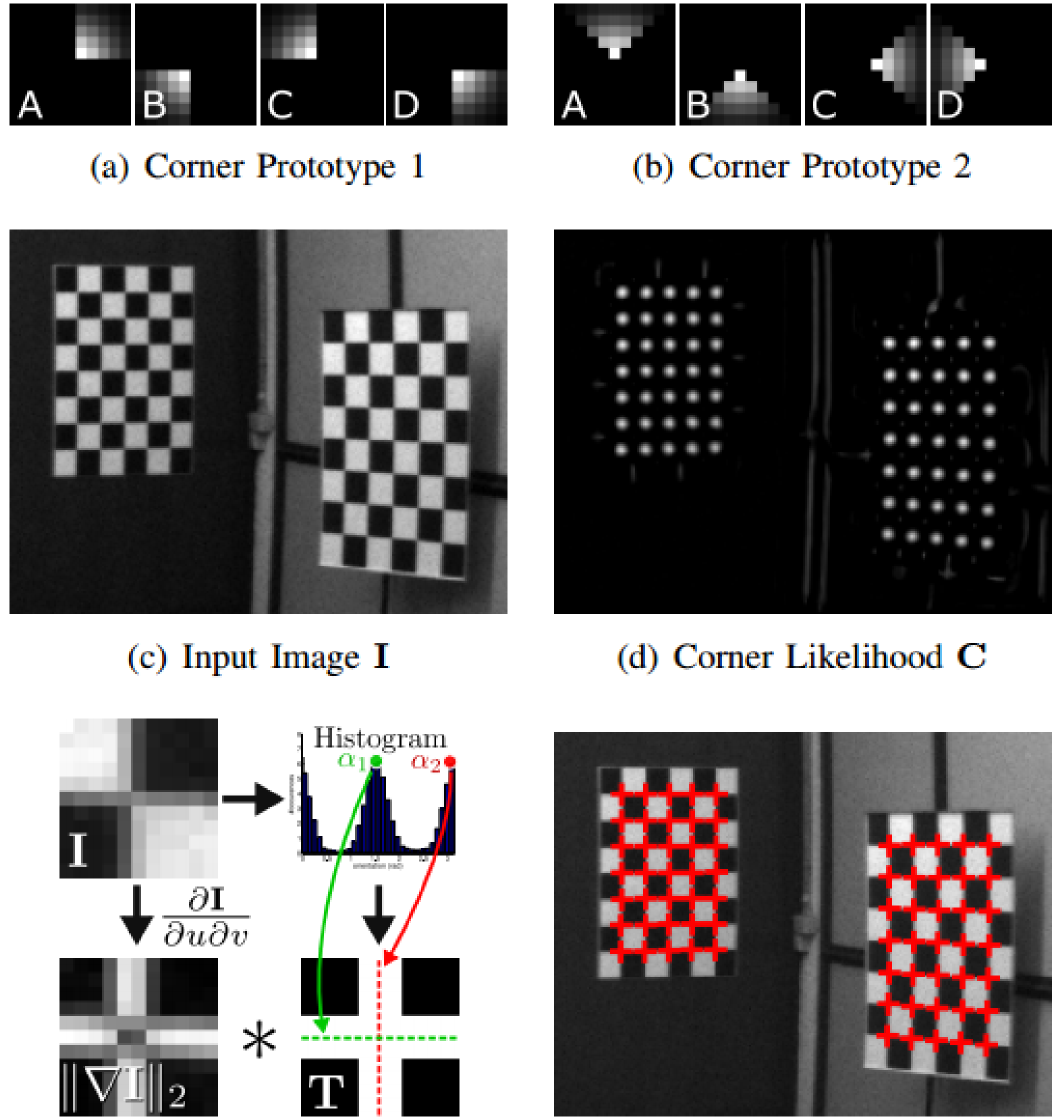
- This is how OpenCV's "findChessboardCorners" works
- Doesn't work so well with actual game images





Approach: Look for corners

- Convolve image with templates for the corners
- Fit to a grid
- Matlab's "detectCheckerboardPoints" function



(a) Corner Prototype 1 (b) Corner Prototype 2

(c) Input Image **I** (d) Corner Likelihood **C**

(e) Orientation & Score (f) Detected Checkerboard Corners

Fig. 2. **Corner detection.** We filter the input image **I** using corner prototypes, apply non-maxima-suppression on the resulting corner likelihood **C** and verify corners by their gradient distribution. See Sec. III-A for details.

Geiger, Andreas, et al. "Automatic camera and range sensor calibration using a single shot." *Robotics and Automation (ICRA), 2012 IEEE International Conference on*. IEEE, 2012.

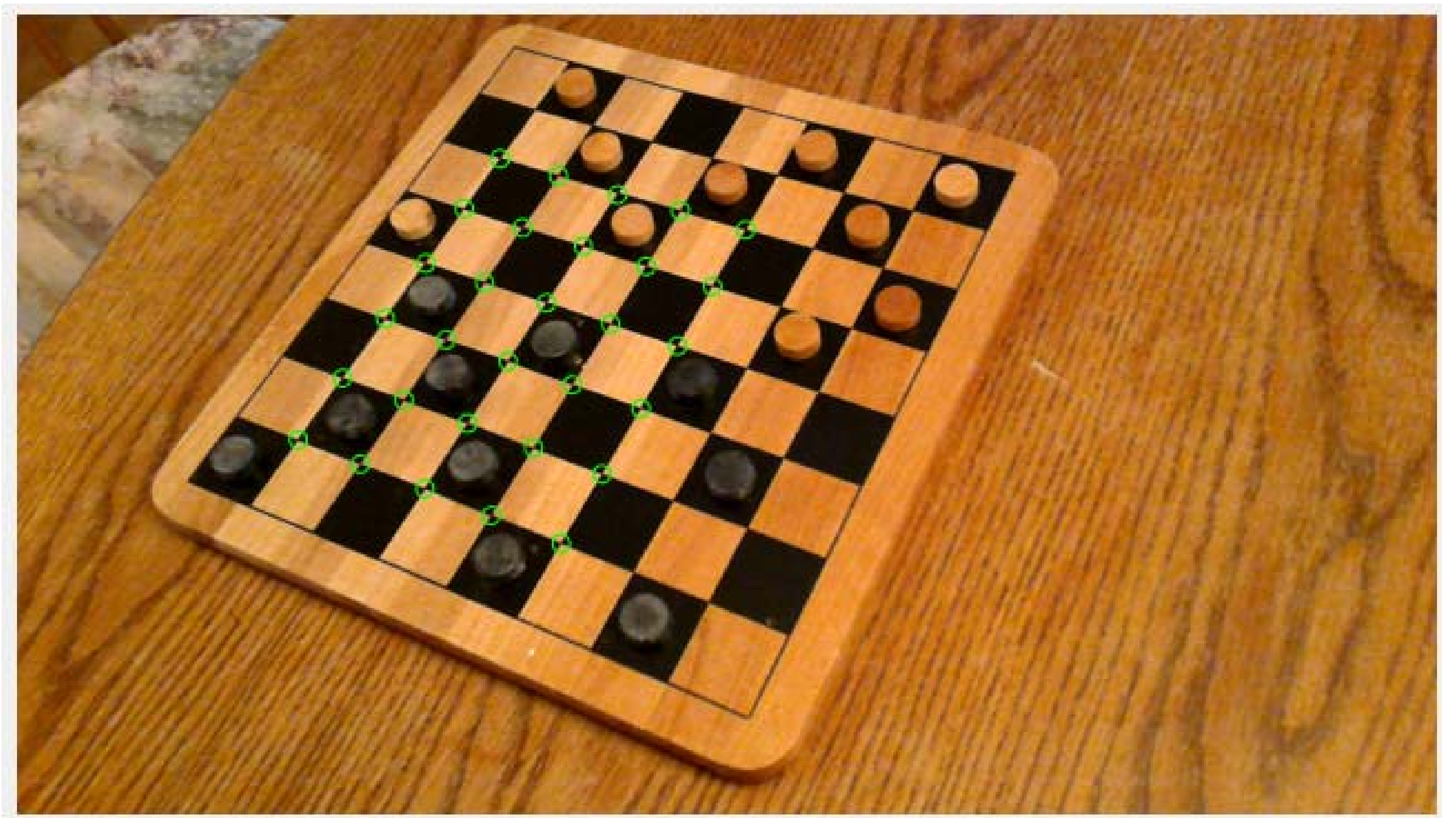
Approach: Look for the corners

Example use of Matlab's "detectCheckerboardPoints"

```
clear variables
close all

% Open movie file.
movieObj = VideoReader('checkers2.mp4');

nFrames = movieObj.NumberOfFrames;
fprintf('Opening movie file with %d images\n', nFrames);

% Go through movie.  We don't need to process every frame.
for iFrame=1:10:nFrames
    I = read(movieObj,iFrame);
    fprintf('Frame %d\n', iFrame);

    % Reduce image size; is faster and we don't need full size to find board.
    if size(I,2)>640
        I = imresize(I, 640/size(I,2));
    end

    figure(1), imshow(I), title(sprintf('Frame %d', iFrame));

    [imagePoints,boardSize] = detectCheckerboardPoints(I);
    hold on; plot(imagePoints(:,1), imagePoints(:,2), 'go');

    pause(0.1);
end
```

Approach: Look for the corners

- Works better … but if any corners are missed, the whole pattern is rejected



Approach: Hough Lines

- Use Hough transform to find long lines
- Then try to match the detected lines to the known model, consisting of a set of 9x9 lines
- Note that the image of the board can be mapped to a reference image via a homography
- This helps to verify that the true lines are found

Matlab Code

- Enter the Matlab code on the next couple of pages
  - A main program, save it as "main.m"
  - A function, save it as "findCheckerBoard.m"
- Get the test video called "board.mp4"

- Run the code - it should read every 10^{th} image and detect edges

Main program

```
clear variables
close all

% Open movie file.
movieObj = VideoReader('board.mp4');

nFrames = movieObj.NumberOfFrames;
fprintf('Opening movie file with %d images\n', nFrames);

% Go through movie.  We don't need to process every frame.
for iFrame=1:10:nFrames
    I = read(movieObj,iFrame);
    fprintf('Frame %d\n', iFrame);

    % Reduce image size; is faster and we don't need full size to find board.
    if size(I,2)>640
        I = imresize(I, 640/size(I,2));
    end

    figure(1), imshow(I), title(sprintf('Frame %d', iFrame));

    % Find the checkerboard.  Return the four outer corners as a 4x2 array,
    % in the form [ [x1,y1]; [x2,y2]; ... ].
    [corners, nMatches, avgErr] = findCheckerBoard(I);


    pause;
end
```

Function “findCheckerBoard”

```
function [corners, nMatches, avgErr] = findCheckerBoard(I)
% Find a 8x8 checkerboard in the image I.
% Returns:
%   corners: the locations of the four outer corners as a 4x2 array, in
%       the form [ [x1,y1]; [x2,y2]; ... ].
%   nMatches:  number of matching points found (ideally is 81)
%   avgErr:  the average reprojection error of the matching points
% Return empty if not found.

corners = [];
nMatches = [];
avgErr = [];

if size(I,3)>1
    I = rgb2gray(I);
end

% Do edge detection.
E = edge(I, 'canny');
figure(10), imshow(E), title('Edges');

end
```

Look at Edge Output Images

- There are too many edges – we only need the edge points on the board, not all the ones in the background
- The edges on the board should be relatively strong
- Raise Canny threshold and run again
  - Replace

```
E = edge(I, 'canny');
```

  - with

```
[~,thresh] = edge(I, 'canny');      % First get the automatic threshold
E = edge(I, 'canny', 5*thresh);     % Raise the threshold
```

Hough Transform

- Add this code to do the Hough transform on the edge image E and extract peaks

```
% Do Hough transform to find lines.
[H,thetaValues,rhoValues] = hough(E);

% Extract peaks from the Hough array H.  Parameters for this:
%   houghThresh:  Minimum value to be considered a peak.  Default
%       is 0.5*max(H(:))
%   NHoodSize:  Size of suppression neighborhood.  Default is
%       [size(H,1)/50, size(H,2)/50].  Must be odd numbers.
myThresh = ceil(0.5*max(H(:)));
NHoodSize = ceil([size(H,1)/50, size(H,2)/50]);
% Force odd size
if mod(NHoodSize(1),2)==0  NHoodSize(1) = NHoodSize(1)+1;  end
if mod(NHoodSize(2),2)==0  NHoodSize(2) = NHoodSize(2)+1;  end
peaks = houghpeaks(H, ...
    30, ...                 % Maximum number of peaks to find
    'Threshold', myThresh, ...      % Threshold for peaks
    'NHoodSize', NHoodSize);    % Default = floor(size(H)/50);
```

Display lines

- Add this code to mark the peaks on the Hough array

```
% Display Hough array and draw peaks on Hough array.
figure(11), imshow(H, []), title('Hough'), impixelinfo;
for i=1:size(peaks,1)
    rectangle('Position', ...
        [peaks(i,2)-NHoodSize(2)/2, peaks(i,1)-NHoodSize(1)/2, ...
        NHoodSize(2), NHoodSize(1)], 'EdgeColor', 'r');
end
```

- Add this code to display all lines. This calls a function "drawLines" to draw lines on the edge image

```
% Show all lines.
figure(10), imshow(E), title('All lines');
drawLines( ...
    rhoValues(peaks(:,1)), ...      % rhos for the lines
    thetaValues(peaks(:,2)), ...    % thetas for the lines
    size(E), ...                    % size of image being displayed
    'y');                           % color of line to display
```

- Also, add the function "drawLines" on the next page, at the end of file "findCheckerBoard".

Function "drawLines"

```
function drawLines(rhos, thetas, imageSize, color)
% This function draws lines on whatever image is being displayed.
% Input parameters:
%   rhos,thetas: representation of the line (theta in degrees)
%   imageSize:  [height,width] of image being displayed
%   color:  color of line to draw

% Equation of the line is rho = x cos(theta) + y sin(theta), or
%    y = (rho - x*cos(theta))/sin(theta)

for i=1:length(thetas)
    if abs(thetas(i)) > 45
        % Line is mostly horizontal.  Pick two values of x,
        % and solve for y = (-ax-c)/b
        x0 = 1;
        y0 = (-cosd(thetas(i))*x0+rhos(i))/sind(thetas(i));
        x1 = imageSize(2);
        y1 = (-cosd(thetas(i))*x1+rhos(i))/sind(thetas(i));
    else
        % Line is mostly vertical. Pick two values of y,
        % and solve for x = (-by-c)/a
        y0 = 1;
        x0 = (-sind(thetas(i))*y0+rhos(i))/cosd(thetas(i));
        y1 = imageSize(1);
        x1 = (-sind(thetas(i))*y1+rhos(i))/cosd(thetas(i));
    end

    line([x0 x1], [y0 y1], 'Color', color);
    text(x0,y0,sprintf('%d', i), 'Color', color);
end

end
```

Hough Transform

- Look at detected lines. Some important ones aren't detected.
- Too few edge points on those lines … peaks are too low.
- Lower Hough peak threshold – change

```
myThresh = ceil(0.5*max(H(:)));
```

- To

```
myThresh = ceil(0.05*max(H(:)));
```

- Verify that important lines are now detected.

“Orthogonal” Lines

- Now find the two (approximately orthogonal) sets of lines.
- We’ll search for the two largest peaks in the histogram of line angles.
  - (Note – a better way is to find the two “vanishing points” … see Szeliski book section 4.3.3)
- Keep only those lines that are near the angles corresponding to the two largest peaks
- Enter the code on the next few pages to find the lines and show them

“Orthogonal” Lines

- This goes just after finding the code to display all the lines.
  - It calls a function “findOrthogonalLines” (see next page)

```
% Find two sets of orthogonal lines.
[lines1, lines2] = findOrthogonalLines( ...
    rhoValues(peaks(:,1)), ...      % rhos for the lines
    thetaValues(peaks(:,2)));       % thetas for the lines


% Show the two sets of lines.
figure(12), imshow(E), title('Orthogonal lines');
drawLines( ...
    lines1(2,:), ...    % rhos for the lines
    lines1(1,:), ...    % thetas for the lines
    size(E), ...        % size of image being displayed
    'g');               % color of line to display
drawLines( ...
    lines2(2,:), ...    % rhos for the lines
    lines2(1,:), ...    % thetas for the lines
    size(E), ...        % size of image being displayed
    'r');               % color of line to display
```

Function
“findOrthogonalLines”
(1 of 2)

Put this at the end of the file “findCheckerBoard.m”

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Find two sets of orthogonal lines.
% Inputs:
%   rhoValues:  rho values for the lines
%   thetaValues:  theta values (should be from -90..+89 degrees)
% Outputs:
%   lines1, lines2:  the two sets of lines, each stored as a 2xN array,
%       where each column is [theta;rho]
function [lines1, lines2] = findOrthogonalLines( ...
    rhoValues, ...      % rhos for the lines
    thetaValues)        % thetas for the lines

% Find the largest two modes in the distribution of angles.
bins = -90:10:90;       % Use bins with widths of 10 degrees
[counts, bins] = histcounts(thetaValues, bins);    % Get histogram
[~,indices] = sort(counts, 'descend');

% The first angle corresponds to the largest histogram count.
a1 = (bins(indices(1)) + bins(indices(1)+1))/2;     % Get first angle

% The 2nd angle corresponds to the next largest count.  However, don't
% find a bin that is too close to the first bin.
for i=2:length(indices)
    if (abs(indices(1)-indices(i)) <= 2) || ...
            (abs(indices(1)-indices(i)+length(indices)) <= 2) || ...
            (abs(indices(1)-indices(i)-length(indices)) <= 2)
        continue;
    else
        a2 = (bins(indices(i)) + bins(indices(i)+1))/2;
        break;
    end
end
fprintf('Most common angles: %f and %f\n', a1, a2);
```

Function “findOrthogonalLines” (1 of 2)

```
% Get the two sets of lines corresponding to the two angles.  Lines will
% be a 2xN array, where
%   lines1[1,i] = theta_i
%   lines1[2,i] = rho_i
lines1 = [];
lines2 = [];
for i=1:length(rhoValues)
    % Extract rho, theta for this line
    r = rhoValues(i);
    t = thetaValues(i);

    % Check if the line is close to one of the two angles.
    D = 25;     % threshold difference in angle
    if abs(t-a1) < D || abs(t-180-a1) < D || abs(t+180-a1) < D
        lines1 = [lines1 [t;r]];
    elseif abs(t-a2) < D || abs(t-180-a2) < D || abs(t+180-a2) < D
        lines2 = [lines2 [t;r]];
    end
end

end
```

Sorting Lines

- Sort lines from top to bottom, left to right
- Strategy:
  - if lines are mostly horizontal, sort on vertical position.
  - If lines are mostly vertical, sort on horizontal position.
- Insert this code just after the call to "findOrthogonalLines"
  - It calls "sortLines" (on the next page)

```
% Sort the lines, from top to bottom (for horizontal lines) and left to
% right (for vertical lines).
lines1 = sortLines(lines1, size(E));
lines2 = sortLines(lines2, size(E));
```

- Note that indices are (almost) in order now.

Function “sortLines”

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Sort the lines.
% If the lines are mostly horizontal, sort on vertical distance from yc.
% If the lines are mostly vertical, sort on horizontal distance from xc.
function lines = sortLines(lines, sizeImg)

xc = sizeImg(2)/2;  % Center of image
yc = sizeImg(1)/2;

t = lines(1,:);     % Get all thetas
r = lines(2,:);     % Get all rhos

% If most angles are between -45 .. +45 degrees, lines are mostly
% vertical.
nLines = size(lines,2);
nVertical = sum(abs(t)<45);
if nVertical/nLines > 0.5
    % Mostly vertical lines.
    dist = (-sind(t)*yc + r)./cosd(t) - xc;  % horizontal distance from center
else
    % Mostly horizontal lines.
    dist = (-cosd(t)*xc + r)./sind(t) - yc;  % vertical distance from center
end

[~,indices] = sort(dist, 'ascend');
lines = lines(:,indices);

end
```

Put this at the end of the file “findCheckerBoard.m”

Find Intersections

- Calculate all possible intersections between the two sets of lines.
- Note – the intersection of two lines can be found as follows (see Szeliski book section 2.1.1)
  - A line is represented by the parameters (*a,b,c*), where the equation of the line is *ax+by+c* = 0
  - If $l_1 = (a_1, b_1, c_1)$ and $l_2 = (a_2, b_2, c_2)$, the point of intersection is the cross product $p = l_1 \times l_2$

```
% Intersect every pair of lines, one from set 1 and one from set 2.
% Output is the x,y coordinates of the intersections:
%   xIntersections(i1,i2): x coord of intersection of i1 and i2
%   yIntersections(i1,i2): y coord of intersection of i1 and i2
[xIntersections, yIntersections] = findIntersections(lines1, lines2);

% Plot all measured intersection points.
hold on
plot(xIntersections(:),yIntersections(:),'yd');
hold off
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Intersect every pair of lines, one from set 1 and one from set 2.
% Output arrays contain the x,y coordinates of the intersections of lines.
%   xIntersections(i1,i2): x coord of intersection of i1 and i2
%   yIntersections(i1,i2): y coord of intersection of i1 and i2
function [xIntersections, yIntersections] = findIntersections(lines1, lines2)
N1 = size(lines1,2);
N2 = size(lines2,2);

xIntersections = zeros(N1,N2);
yIntersections = zeros(N1,N2);
for i1=1:N1
    % Extract rho, theta for this line
    r1 = lines1(2,i1);
    t1 = lines1(1,i1);

    % A line is represented by (a,b,c), where ax+by+c=0.
    % We have r = x cos(t) + y sin(t), or  x cos(t) + y sin(t) - r = 0.
    l1 = [cosd(t1); sind(t1); -r1];

    for i2=1:N2
        % Extract rho, theta for this line
        r2 = lines2(2,i2);
        t2 = lines2(1,i2);

        l2 = [cosd(t2); sind(t2); -r2];

        % Two lines l1 and l2 intersect at a point p where p = l1 cross l2
        p = cross(l1,l2);
        p = p/p(3);

        xIntersections(i1,i2) = p(1);
        yIntersections(i1,i2) = p(2);
    end
end

end
```

Function "findIntersections"

Put this at the end of the file "findCheckerBoard.m"

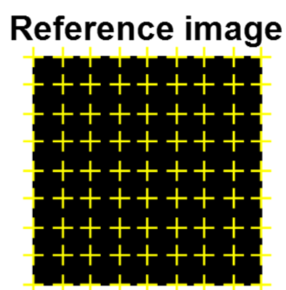
Strategy

- If we can find the four outer lines, their intersections define the outer corners of the board.
- If they are correct, we can predict the intersections of all 9x9 lines.
- We're going to need a reference image that is a model of what we are looking for.
  - Define a reference image of size 100x100

```
% Define a "reference" image.
IMG_SIZE_REF = 100;      % Reference image is IMG_SIZE_REF x IMG_SIZE_REF

% Get predicted intersections of lines in the reference image.
[xIntersectionsRef, yIntersectionsRef] = createReference(IMG_SIZE_REF);
```

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Get predicted intersections of lines in the reference image.
function [xIntersectionsRef, yIntersectionsRef] = createReference(sizeRef)

sizeSquare = sizeRef/8;     % size of one square

% Predict all line intersections.
[xIntersectionsRef, yIntersectionsRef] = meshgrid(1:9, 1:9);
xIntersectionsRef = (xIntersectionsRef-1)*sizeSquare + 1;
yIntersectionsRef = (yIntersectionsRef-1)*sizeSquare + 1;

% Draw reference image.
Iref = zeros(sizeRef+1, sizeRef+1);
figure(13), imshow(Iref), title('Reference image');

% Show all reference image intersections.
hold on
plot(xIntersectionsRef, yIntersectionsRef, 'y+');
hold off

end
```

Function “createReference”



Put this at the end of the file “findCheckerBoard.m”

Finding Correspondence

- Now, search for correspondences between the points from the input image and the reference image
- Given correspondences of the four points representing the outside corners of the board, we can compute a homography between the input image and the reference image.
  - We can then predict the locations of all interior points.
  - The best fit has the most matches with lowest projection error.

```
% Find the best correspondence between the points in the input image and
% the points in the reference image.  If found, the output is the four
% outer corner points from the image, represented as a a 4x2 array, in the
% form [ [x1,y1]; [x2,y2]; ... ].
[corners, nMatches, avgErr] = findCorrespondence( ...
    xIntersections, yIntersections, ...         % Input image points
    xIntersectionsRef, yIntersectionsRef, ...   % Reference image points
    I);
```

```
% Find the best correspondence between the points in the input image and
% the points in the reference image.  If found, the output is the four
% outer corner points from the image, represented as a a 4x2 array, in the
% form [ [x1,y1]; [x2,y2], ... ].
function [corners, nMatchesBest, avgErrBest] = findCorrespondence( ...
    xIntersections, yIntersections, ...         % Input image points
    xIntersectionsRef, yIntersectionsRef, ...   % Reference image points
    I)

% Get the coordinates of the four outer corners of the reference image,
% in clockwise order starting from the top left.
pCornersRef = [ ...
    xIntersectionsRef(1,1), yIntersectionsRef(1,1);
    xIntersectionsRef(1,end), yIntersectionsRef(1,end);
    xIntersectionsRef(end,end), yIntersectionsRef(end,end);
    xIntersectionsRef(end,1), yIntersectionsRef(end,1) ];

M = 4;      % Number of lines to search in each direction
DMIN = 4;   % To match, a predicted point must be within this distance

nMatchesBest = 0;   % Number of matches of best candidate found so far
avgErrBest = 1e9;   % The average error of the best candidate

N1 = size(xIntersections,1);
N2 = size(xIntersections,2);

for i1a=1:min(M,N1)
    for i1b=N1:-1:max(N1-M,i1a+1)
        for i2a=1:min(M,N2)
            for i2b=N2:-1:max(N2-M,i2a+1)

                % Get the four corners corresponding to the intersections
                % of lines (1a,2a), (1a,2b), (1b,2b, and (1b,2a).
                pCornersImg = zeros(4,2);
                pCornersImg(1,:) = [xIntersections(i1a,i2a) yIntersections(i1a,i2a)];
                pCornersImg(2,:) = [xIntersections(i1a,i2b) yIntersections(i1a,i2b)];
                pCornersImg(3,:) = [xIntersections(i1b,i2b) yIntersections(i1b,i2b)];
                pCornersImg(4,:) = [xIntersections(i1b,i2a) yIntersections(i1b,i2a)];

                % Make sure that points are in clockwise order.
                % If not, exchange points 2 and 4.
```

Function “findCorrespondence” (1 of 3)

Put this at the end of the file “findCheckerBoard.m”

Function "findCorrespondence" (2 of 3)

```
v12 = pCornersImg(2,:) - pCornersImg(1,:);
v13 = pCornersImg(3,:) - pCornersImg(1,:);
if v12(1)*v13(2) - v12(2)*v13(1) < 0
    temp = pCornersImg(2,:);
    pCornersImg(2,:) = pCornersImg(4,:);
    pCornersImg(4,:) = temp;
end


% Fit a homography using those four points.
T = fitgeotrans(pCornersRef, pCornersImg, 'projective');

% Transform all reference points to the image.
pIntersectionsRefWarp = transformPointsForward(T, ...
    [xIntersectionsRef(:) yIntersectionsRef(:)]);


% For each predicted reference point, find the closest
% detected image point.
dPts = 1e6 * ones(size(pIntersectionsRefWarp,1),1);
for i=1:size(pIntersectionsRefWarp,1)
    x = pIntersectionsRefWarp(i,1);
    y = pIntersectionsRefWarp(i,2);
    d = ((x-xIntersections(:)).^2 + (y-yIntersections(:)).^2).^0.5;
    dmin = min(d);
    dPts(i) = dmin;
end

% If the distance is less than DMIN, count it as a match.
nMatches = sum(dPts < DMIN);

% Calculate the avg error of the matched points.
avgErr = mean(dPts(dPts < DMIN));

% Keep the best combination found so far, in terms of
% the number of matches and the minimum error.
if nMatches < nMatchesBest
    continue;
end
if (nMatches == nMatchesBest) && (avgErr > avgErrBest)
    continue;
end
```

Function "findCorrespondence" (3 of 3)

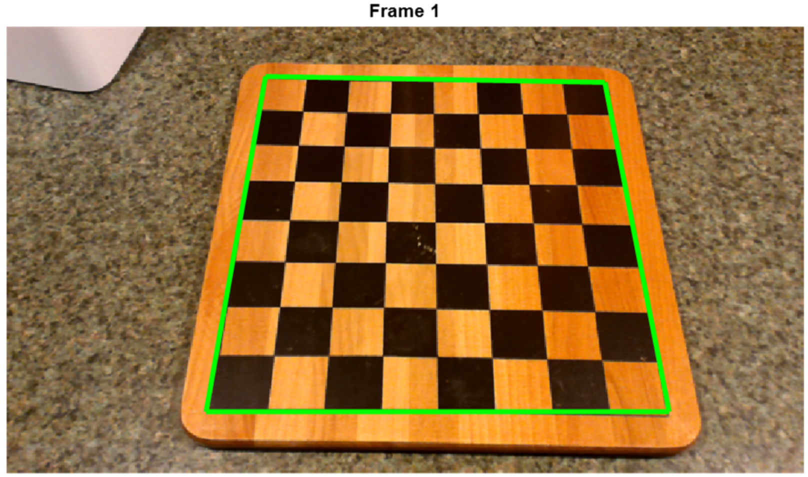
```
                % Got a better combination; save it.
                avgErrBest = avgErr;
                nMatchesBest = nMatches;
                corners = pCornersImg;

                % Display the predicted and measured points.
                figure(14), imshow(I,[]);
                title('Predicted and measured points');
                hold on
                plot(xIntersections(:), yIntersections(:), 'g.');
                plot(pIntersectionsRefWarp(:,1), pIntersectionsRefWarp(:,2), 'yo');
                hold off

                rectangle('Position', [pCornersImg(1,1)-10 pCornersImg(1,2)-10 20 20], ...
                    'Curvature', [1 1], 'EdgeColor', 'r', 'LineWidth', 2);
                rectangle('Position', [pCornersImg(2,1)-10 pCornersImg(2,2)-10 20 20], ...
                    'Curvature', [1 1], 'EdgeColor', 'g', 'LineWidth', 2);
                rectangle('Position', [pCornersImg(3,1)-10 pCornersImg(3,2)-10 20 20], ...
                    'Curvature', [1 1], 'EdgeColor', 'b', 'LineWidth', 2);
                rectangle('Position', [pCornersImg(4,1)-10 pCornersImg(4,2)-10 20 20], ...
                    'Curvature', [1 1], 'EdgeColor', 'y', 'LineWidth', 2);
                fprintf(' Found %d matches, average error = %f\n', ...
                    nMatchesBest, avgErrBest);

                pause
            end
        end
    end
end

end
```

Displaying the Board

```
% Find the checkerboard.  Return the four outer corners as a 4x2 array,
% in the form [ [x1,y1]; [x2,y2]; ... ].
   [corners, nMatches, avgErr] = findCheckerBoard(I);
```

- In the main program, check the number of matches returned by "findCheckerBoard".
  - The ideal number is 81.
  - If the number found is much less than this, the board was probably not found.
- Then you can draw lines around the four outer corners.

Frame 1



Displaying the Board

- Convert the image of the board to an “orthophoto”.
- Define the ideal corners in the orthophoto:

```
% Define the outside corners for a square "reference" image, size LxL.
cornersRef = [ 1,1; L,1; L,L; 1,L ];
```

- Call fitgeotrans to compute the homography:

```
% Fit a projective transform that will map image to reference.
T = fitgeotrans(corners, cornersRef, 'projective');
```

- Then call “imwarp” to warp the input image to the output orthophoto:

```
% Create an "orthophoto" of the image of the board.
Iboard = imwarp(I, T, 'OutputView', imref2d([L L], [1 L], [1 L]));
```