Interactive Educational Tool:
Depletion of Fully Penetrating Stream by a Pumping Well

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Organization Background:
The Groundwater Project was initiated in 2017 to enhance prospects for expansion of human capacity in groundwater problem solving and public understanding of groundwater. The Project is a volunteer-based, nonprofit, nongovernmental organization (NGO) registered in Canada and global in scope. It advances groundwater understanding and awareness by creating high-quality learning materials in many languages and making them available for download free-of-charge from its website. The GW-Project is led by Dr. John Cherry (recipient of the 2020 Stockholm Water Prize) and functions with a two-person staff, a volunteer board of directors, and over 1000 volunteer authors, reviewers, editors, and translators.

Description of Project:
Create an online, interactive, educational tool to demonstrate depletion of flow from a fully penetrating stream in response to a pumping well. You can build the tool from scratch or by modifying the files for other tools that I provide below. To help you understand what I mean by an interactive tool, visit this example of a high-level tool at tdpfonline.net. The TopoDrive tool is more complex than what I expect you will create in this field session. There are written directions for how to use it in the header menu but you may prefer to view this video that explains what to do after clicking on Launch TopoDrive. On the other end of the spectrum is a much simpler tool called Water Table Recharge (WTR) a beta version is online at https://interactive-education.gw-project.org/wtr/. A video showing how WTR is used is provided at that link. I provide links to the complete set of files for both tools in the section on “Tasks” below. I submitted two projects with different options. To help you figure out which you prefer, the difference between the proposals is the underlined text and the “Details of the Tool” sections.

Tasks:

Warm-up:

- To get a feel for the underlying files of an interactive learning tool, review the HTML and JavaScript files for the existing tools (WTR and TDPF) that you are welcome to use as a base or source of code for your work. You can view and try them out by retrieving the zip files from my DropBox links below, unzipping them on your computer, and double clicking on index.html.

  - WTR.zip contains the current Water Table Recharge (WTR) tool. It is a simple JavaScript code that I wrote recently while teaching myself to code in JavaScript. I welcome your advice on improving the code. It will be updated and finalized by the time you begin the project and I will provide updates as needed at that time.
  - TDPF.zip contains the Topodrive/ParticleFlow (TDPF) tool. It is a more complex code relying on the object-oriented programming features of JavaScript. This was created by a past Groundwater Project board member, Paul Hsieh, who moved on to other endeavors. It is public domain, housed on one of his websites (tdpfonline.net) and referenced from a couple of Groundwater Project books.
- When building the tool for this project, please use object-oriented programing as in the TDPF tool to code the different views of results not the linear structure of the WTR tool.
- Any time the team meets an obstacle, please contact me soon so we can brainstorm a solution or an alternative approach.

Building the Tool:

- The crux of your work is to develop the files that provide input to the online learning tool, make calculations, and produce graphical output. I suggest you use the letters DFPS as an acronym for "Depletion Fully Penetrating Stream" in the file names. I will use DFPS for shorthand in the following discussion. The crux of the WTR tool is wrt.html and associated JavaScript file (wtr.js) and the crux of the TD tool is topodrive.html and a number of associated JavaScript files (td*.js).
  - Write/modify the *.html and *.js files for the user to input the required data for DFPS.
  - Write the JavaScript needed to make the DFPS calculations. I provide the equations and parameter definitions in the "Details of the Tool" section below. Note: if you are unfamiliar with any of the functions in the mathematics below, I provide simple expressions for you to use and will give you a spreadsheet solution so you can compare the results to your code.
  - Write the HTML and JavaScript code lines to pass the DFPS results from the JavaScript to the HTML file and present the results as text, maps, and graphs.
  - Consider your tool from the perspective of an uninformed user and enhance as needed to help them intuit how to use it.
  - Try to find inputs that "break" the tool and include code to catch those problems and write error or warning messages to the user and deal with the problem gracefully in the output. I can help with issues related to results that may look reasonable in numbers and graphs but that do not make physical sense.
- After building the tool we will need the surrounding HTML files to present information describing the tool. The existing files for WTR will be modified to fit DFPS. These files

are the index.html, about.html, theory-WTR.html, example-WTR.html, how-to-use-WTR.html, exercise-WTR.html, and further-exploration-WTR.html. I will do the necessary work on these files, but I welcome your contributions and suggestions. I will need information about the team members for the about.html file.

- The final product will be included on the Groundwater Project web site gw-project.org. A menu item for Interactive Education will be added to that site soon.
- The WTR and TDPF files I provided include calculation of values and creation of graphs and contour maps so even if you do not edit those files directly to create your tool, they will be a useful source of information.

Details of the Tool:

Input Parameters for the user input form

Spatial dimensions

- Perpendicular distance from the well to the stream, d, in meters
- Factor for size of domain (Fd) to be displayed, F is dimensionless (the mathematical domain is infinite but the tool will provide a square of specified size in meters)

Aquifer parameters

- Hydraulic conductivity K_a in centimeters/second then converted to meters per day
- Aquifer thickness b in meters
- Specific Yield S_y (dimensionless)

Note T is used in the calculations and is calculated as: T=Kb

Stream parameters

- Hydraulic conductivity of stream bed K_{sb} in centimeters/second then converted to meters per day
- Stream flowrate Q_s in cubic meters per second

Well information the well location will be defined as x,y = 0,0

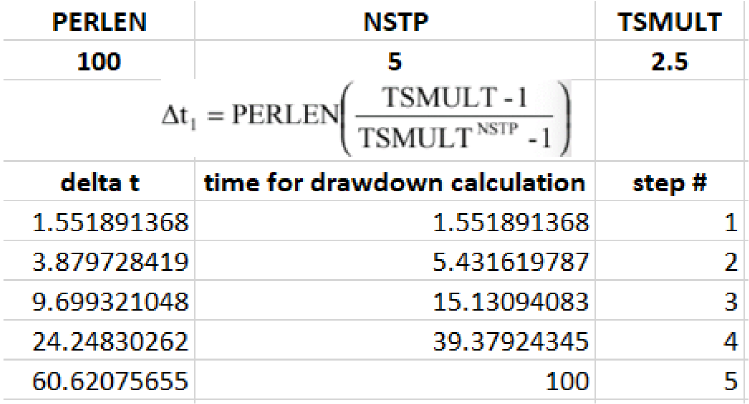
- pumping rate, Q_w in liters per minute then converted to cubic meters per day and sometimes per second for calculations (positive for withdrawal of water and negative for injection)
- locations of observation wells for graphing drawdown as a function of time
    - ox1,oy1 in meters relative to the pumping well (can be + or -)
    - ox2,oy2 in meters relative to the pumping well (can be + or -)

Time information

- Duration of pumping, t in days, always positive
    - Number of increments of time to be calculated (minimum 1 up to some allowed maximum, perhaps 10?)

Prepare Map of Drawdown, s

- The lower left corner of the contour map will be y = -(Fd/2) and x = -(Fd-d).
- Draw a dot for the well at x,y=0,0 and a vertical line for the stream at x=d and from y=-Fd/2 to +Fd/2.
- Calculate the time increments logarithmically Δtime1 = TotalTime ((2.5-1)/(2.5$^{\#\text{increments}}$–1)), each time step there after will be 2.5 times longer and added to the previous cumulative time. You could let the user specify the multiplier. Here is an example.



PERLEN	NSTP	TSMULT
100	**5**	**2.5**
	$\Delta t_1 = \text{PERLEN}\left(\frac{\text{TSMULT} - 1}{\text{TSMULT}^{\text{NSTP}} - 1}\right)$	
delta t	**time for drawdown calculation**	**step #**
1.551891368	1.551891368	1
3.879728419	5.431619787	2
9.699321048	15.13094083	3
24.24830262	39.37924345	4
60.62075655	100	5

- Calculate the fraction of pumped water that is coming from the stream (i.e., stream depletion) at time t as follows (note: use consistent units):

$$Q_{\text{fraction}} = \text{erfc}\left(\sqrt{\frac{Sd^2}{4Tt}}\right)$$

where:

Q_{fraction} = fraction of pumping rate leaking from stream into groundwater system at time t (dimensionless)

S_y = specific yield (dimensionless)

d = perpendicular distance from well to stream in meters

T = transmissivity ($K_a b$) in meters^2 per day

K_a = aquifer hydraulic conductivity (centimeters per second – convert to meters per day)

b = aquifer thickness (meters)

t = time in days

erfc = complimentary error function = 1-erf, approximation of erf given below

erf(β) = a fairly good approximation is $\text{erf}(\beta) \approx \frac{2}{\pi}\arctan\left(2\beta(1+\beta^4)\right)$

- Calculate the rate of leakage from the stream in meters^3 per day at time t then convert to meters^3 per second for the output graphs:

$$Q_{\text{streamleakage@t}} = Q_{\text{fraction}}\, Q_w$$

where:

Q_w = pumping rate converted to meters^3 per second

- Calculate the stream discharge in meters^3 per second at time t:

$$Q_{s@t} = Q_s - Q_{streamleakage@t}$$

where:

Q_s = initial volumetric discharge of the stream (meters^3 per second)

$Q_{s@t}$ = volumetric flow rate in the stream at time t (meters^3 per second)

- Choose a grid spacing for the map view within the square where values of drawdown will be calculated so they can be shown as contours on a map and as a line on a cross-sectional graph – make sure one set of grid points is along the centerline of the square from left to right – perhaps you will always use a grid that divides the square into 21 x 21 points with one at 0 and one at the size the user specifies for the dimensions of the square (Fd), then the 11th row of grid points will always be long the center line with 10 grid squares between it and zero and 10 squares between it and the length of the square.
- For each time increment, at each grid point determine the straight-line distance, r, to the well (i.e., r = the square root of the sum of the difference in the x's squared and the difference in the y's squared) and calculate the drawdown at that grid point using the formula below (note: use consistent units).

$$s(x, y, t) = \frac{Q_w}{4\pi T}\left(W(u) - W(u')\right)$$

where:

$s(x,y,t)$ = drawdown at a location x,y and time t

Q_w = constant well discharge rate

T = transmissivity (Kb)

S = storativity ($S_s b$)

t = time

r = straight-line distance from the well to x,y

$W(u)$ = $-\ln(u) + a0 + a1\,u + a2\,u^2 + a3\,u^3 + a4\,u^4 + a5\,u^5$

u = $\frac{\sqrt{x^2+y^2}}{\frac{4Tt}{S}}$ (dimensionless)

u' = $\frac{\sqrt{(2d-x)^2+y^2}}{\frac{4Tt}{S}}$ (dimensionless)

a0 = -0.57721566

a1 = 0.99999193

a2 = -0.24991055

a3 = 0.05519968

a4 = -0.00976004

a5 = 0.00107857

Calculate Velocity Vectors

Given a set of heads on the four corners of a square with side length equal to L, find the maximum and minimum head on a circle with radius 0.01 around the center point of a unit square. Then calculate the magnitude of the Darcy velocity in the full-size cell as the difference between the max and min head values on the circle divided by (0.02*L) and multiplied by the K_a (aquifer hydraulic conductivity). The direction of the Darcy velocity is the angle associated with the minimum head. Specific steps are given below.

To find the value of head at each point on a 0.01 radius circle, starting with north as zero radians, calculate 73 unit-cell x,y locations as (0.5+0.01*sin(radians),0.5+0.01*cos(radians)) to reach 2π radians (the complete circle) at 5-degree intervals (i.e., 2π/72 radian intervals), and calculate the head at those locations as follows.

$$h_{x,y} = a00 + a10\ x + a01\ y + a11\ x\ y$$

where the "a"-values are based on the heads at the 4 corners of the grid cell:

$$a00 = h_{LowerLeft}$$

$$a10 = h_{LowerRight} - h_{LowerLeft}$$

$$a01 = h_{UpperLeft} - h_{LowerLeft}$$

$$a11 = h_{UpperRight} - h_{UpperLeft} - h_{LowerRight} + h_{LowerLeft}$$

Next, find the maximum and minimum of heads (check that these are 180 degrees -- that is, π radians -- apart and if they are not, then print a warning along with their values – a small offset is not a concern for this graphic display). Then, calculate the velocity vector with magnitude $((h_{max}-h_{min})\ K_a)/(0.02\ L)$ and direction equal to the angle at which the minimum head occurred. I will provide an illustrative spreadsheet.

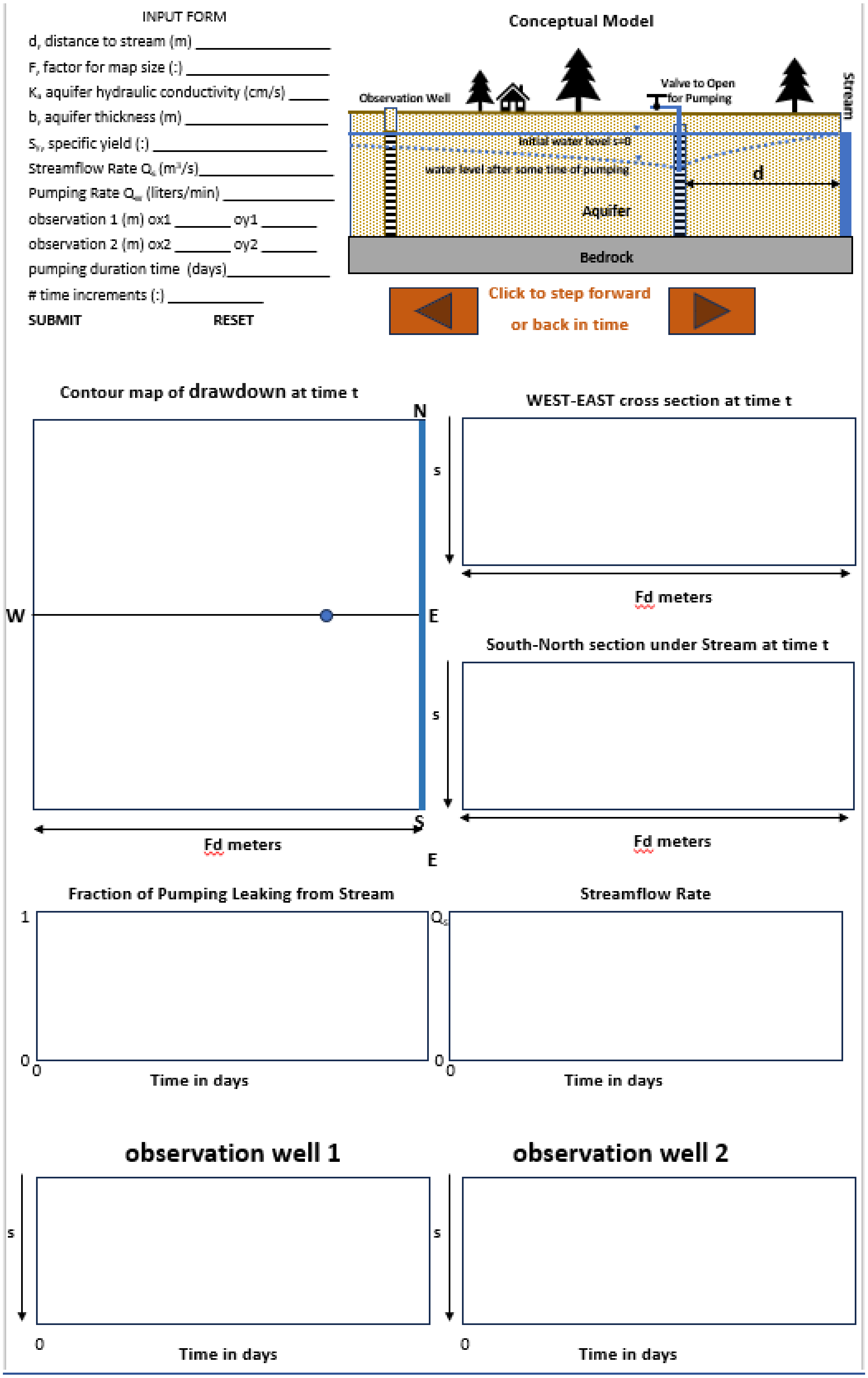
Output to be displayed below the submit and reset button of the input form:

- I include an image of the data input form and output at the end of this document with an example result sketched below it. I will provide the PowerPoint version of the image.
- At each time when the user clicks forward or back, for that time t, draw the **Contour map** of the values of drawdown within the square with a symbol for the location of the pumping well and each observation location o1 o2. If a location falls outside of the square print a message stating that, but continue with calculations to create the contour map within the square.

- Also at each time, draw the **Vector** on each cell, or if that is too crowded, draw vectors on every other, or every third or fourth cell. Adjust the size so the largest vector is scaled to the length of the side of a cell and all other vectors lengths are scaled logarithmically (that is, draw vectors that are the log(calculated value). We use a logarithmic scale because a linear scale may make the smallest vectors too small to see.
- At each time when the user clicks forward or back, for that time t, draw the **Cross-Sections** of drawdown along the mid-line of the square on the W-E and beneath the stream in the S-N direction, that is, for W-E from 0,L/2 to L,L/2 and for S-N at x=d from y=+Fd/2 to -Fd/2.
- At each time when the user clicks forward or back, add/delete the point for that time on the four **Graphs** (Fraction of Pumping, Streamflow Rate, and drawdown versus time for both observation locations.

Student Team Requirements:

- Team size: estimated to be 4 students
- I submitted two similar projects. I hope both will be selected and I encourage the teams to meet a few times to share ideas and solutions in order to make your task easier and the products better.
- Work will be done remotely
- One meeting per week in person or by zoom to review progress, discuss issues (questions are welcome at any time by text, email, or phone)
- Some skill in web site development utilizing HTML and JavaScript
- Interest in learning more JavaScript while undertaking the project and helping me improve my JavaScript skills by learning from you
- My expectation is that we will work together throughout the semester so I can offer what I know to help you and I can learn along with you as you develop another tool
- The Groundwater Project is interested in contracting for development of additional tools



INPUT FORM

d, distance to stream (m) ________________

F, factor for map size (:) ________________

K_a, aquifer hydraulic conductivity (cm/s) _____

b, aquifer thickness (m) ________________

S_y, specific yield (:) ____________________

Streamflow Rate Q_s (m^3/s)__________________

Pumping Rate Q_w (liters/min) ______________

observation 1 (m) ox1 _______ oy1 _______

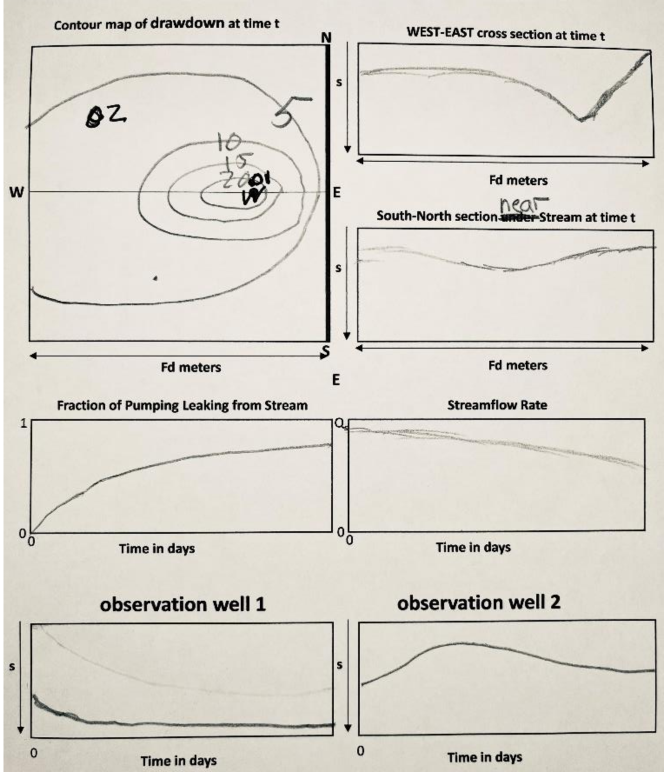
observation 2 (m) ox2 _______ oy2 _______

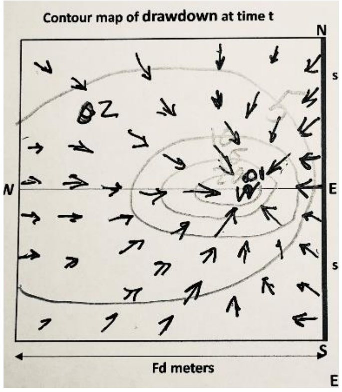
pumping duration time (days)_______________

time increments (:) ___________

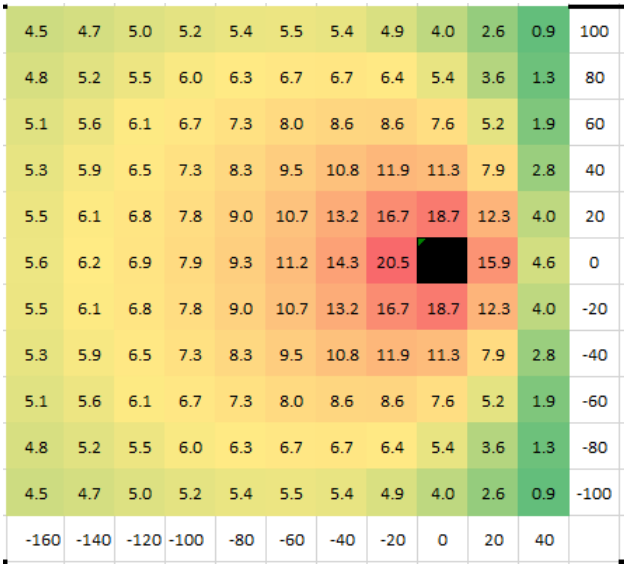
SUBMIT **RESET**

ROUGH HAND SKETCH OF HOW ONE SET OF OUTPUT COULD LOOK
Everything is approximate – inset on right illustrates vectors on contour map





YOU MAY WANT TO USE A COLOR SCALE INSTEAD OF CONTOUR LINES
it would need a legend



4.5	4.7	5.0	5.2	5.4	5.5	5.4	4.9	4.0	2.6	0.9	100
4.8	5.2	5.5	6.0	6.3	6.7	6.7	6.4	5.4	3.6	1.3	80
5.1	5.6	6.1	6.7	7.3	8.0	8.6	8.6	7.6	5.2	1.9	60
5.3	5.9	6.5	7.3	8.3	9.5	10.8	11.9	11.3	7.9	2.8	40
5.5	6.1	6.8	7.8	9.0	10.7	13.2	16.7	18.7	12.3	4.0	20
5.6	6.2	6.9	7.9	9.3	11.2	14.3	20.5		15.9	4.6	0
5.5	6.1	6.8	7.8	9.0	10.7	13.2	16.7	18.7	12.3	4.0	-20
5.3	5.9	6.5	7.3	8.3	9.5	10.8	11.9	11.3	7.9	2.8	-40
5.1	5.6	6.1	6.7	7.3	8.0	8.6	8.6	7.6	5.2	1.9	-60
4.8	5.2	5.5	6.0	6.3	6.7	6.7	6.4	5.4	3.6	1.3	-80
4.5	4.7	5.0	5.2	5.4	5.5	5.4	4.9	4.0	2.6	0.9	-100
-160	-140	-120	-100	-80	-60	-40	-20	0	20	40	