CSCI 262
Data Structures

8 – Recursion

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RECURSION BASICS

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Recursion

Recursion is defining something in terms of itself.

- We define many data structures recursively
  - A linked list node contains a pointer to a node
  - A binary tree node contains two pointers to nodes
- Many functions can be defined recursively:
  - Factorial: $n! = n(n-1)!$
  - Differentiation (chain rule): $\frac{df}{dx} = \frac{df}{dg} \cdot \frac{dg}{dx}$
  - The binomial coefficient: $\binom{n}{k} = \binom{n-1}{k-1} + \binom{n-1}{k}$
- Euclid's algorithm for GCD is recursive!

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Recursive Functions in C++

- Most modern programming languages allow recursion in functions;
- In C++, you simply call a function from within itself, e.g.:

```
unsigned int factorial(unsigned int n) {
   if (n == 0) return 1;
   return n * factorial(n-1);
}
```

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The Base Case

Note the first line of the `factorial` function:

```
unsigned int factorial(unsigned int n) {
   if (n == 0) return 1;
   return n * factorial(n-1);
}
```

What would happen without that line?

When the input n is 0 we call it the *base case*.

The test for the base case **must** come **before** the recursive call!

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RECURSION AND THE STACK

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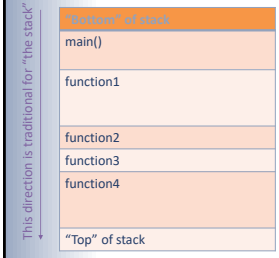
"The Stack" Revisited

When we talk about "the stack", we usually mean a very specific stack; the memory stack of a running program:



Local variables declared in main, return address, other stuff.

Local variables declared in function1; arguments passed by value into function1, return address, other stuff.

Etc.

Each "frame" is created when the function is called, and destroyed when the function exits.

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Recursion and the Stack

Key to understanding recursion in C++:

- Each function *call*, not each function, gets an entry on the stack
  - Each stack entry has memory specific to where we are in the recursion – arguments passing *down*
  - Also need to think about values going *up* as we unwind the stack

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Example: Factorial Start

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```

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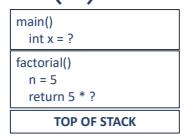
Factorial First Call
Push `factorial(5)`

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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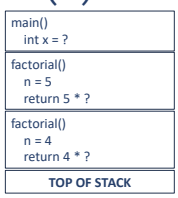
Factorial Second Call
Push `factorial(4)`

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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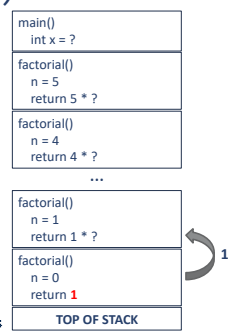
Factorial Sixth Call
Push `factorial(0)` – Base case

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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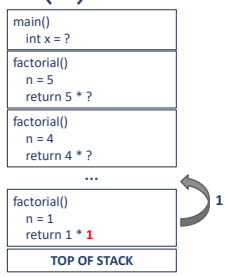
Factorial Unwinding: Pop factorial(0)

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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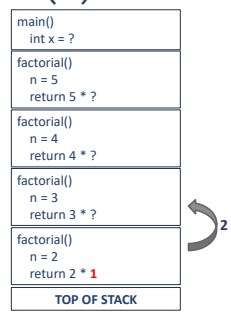
Factorial Unwinding: Pop factorial(1)

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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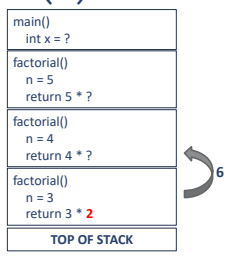
Factorial Unwinding: Pop factorial(2)

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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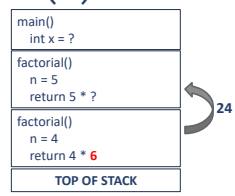
Factorial Unwinding: Pop factorial(3)

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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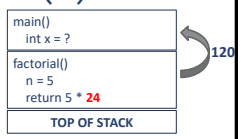
Factorial Unwinding: Pop factorial(4)

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```



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Factorial Unwinding: Pop factorial(5)

```
unsigned factorial(unsigned n) {
    if (n == 0) return 1;
    return n * factorial(n-1);
}

int main() {
    int x = factorial(5);
}
```

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MORE RECURSIVE EXAMPLES

Example: Palindrome

- A palindrome is a recursive object; it is:
  - Empty, or
  - A single character, or
  - A palindrome between two of the same character

Base cases

kayak
civic
redivider
etc.

- Here's a recursive test function:

Sometimes need a recursive *helper* function

```
bool is_palindrome(const string &s, int start, int end) {
    if (end <= start) return true;

    return (s[start] == s[end] && is_palindrome(s, start+1, end-1));
}

bool is_palindrome(const string &s) {
    return is_palindrome(s, 0, s.length() - 1);
}
```


Example: Binomial Coefficient

```
unsigned int nchoosek(unsigned int n, unsigned int k) {
      assert(n >= k);
      if (k == 0 || k == n) return 1;

      return nchoosek(n-1,k) + nchoosek(n-1,k-1);
}
```

Note - more than one base case!

Note - two recursive calls!

Common Mistakes

- No base case:

```
void infinite(int n) {
   cout << n << endl;
   infinite(n-1);
}
```

- Recursion step doesn't reduce problem:

```
void infinite2(int n) {
   if (n < 0) return;
   cout << n << endl;
   infinite2(n);
}
```


Recursion vs. Iteration

Recursion is often the simplest approach.

However, recursion can usually be replaced by iteration plus some storage for intermediate results.

```
unsigned int factorial(unsigned int n) {
   unsigned int ans = 1;
   for (int j = n; j > 1; j--) ans = ans * j;
   return ans;
}
```


Up Next

- Wednesday, February 6
  - More recursion: thinking recursively, backtracking, minimax
- Friday, February 8
  - Lab 5 - TBD
  - Project 2 due
  - APT 3 assigned
- Week of February 11: Interview Grading