CSCI 262
Data Structures

7

Selection Sort

Introduction to Analysis of Algorithms

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Getting things in order

SORTING

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Sorting

- Input: a list of elements, e.g. integers
- Output: a list of the input elements in sorted order

Why do we study this problem?

- Teaching example
  - Algorithm design
  - Algorithm analysis
- Sorting is also useful for all sorts of applications!

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Selection Sort

- Input: a list of elements, e.g. integers
- Output: a list of the input elements in sorted order

- A simple solution:
  - Find the minimum element in the list
  - Swap it with the first element in the list
  - Sort the sublist following the first element

- This sorting algorithm is named **selection sort**.

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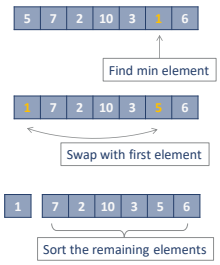
Selection Sort Illustrated



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Selection Sort Code

```
// for vectors of int
void selection_sort(vector<int> &vec) {
    int n = vec.size();
    for (int left = 0; left < n; left++) {
        int right = left;
        for (int j = left + 1; j < n; j++) {
            if (vec[j] < vec[right]) right = j;
        }
        swap(vec[left], vec[right]);
    }
}
```

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Swap Code

```
// exchange two int values
void swap(int &a, int &b) {
    int tmp = a;
    a = b;
    b = tmp;
}
```

This function is actually already available in the standard library; #include <algorithm>

How much?

MEASURING WORK

How Much Work in Selection Sort?

- Difficult to actually count CPU cycles –
  - Differs by CPU
  - Differs by compiler
  - Lots of noise factors: caching, context switching, etc.
- Simplification: just measure comparisons and swaps
  - Ignore loop counter updates, etc.
  - We'll see later why we can get away with this
- Let's count (only somewhat carefully):
  - Use a vector of size 10
  - Later, generalize to size *n*

Analyzing Selection Sort

```
void selection_sort(vector<int> &vec) {
    int n = vec.size();
    for (int left = 0; left < n; left++) {
        int right = left;
        for (int j = left + 1; j < n; j++) {
            if (vec[j] < vec[right]) right = j;
        }
        swap(vec[left], vec[right]);
    }
}
```


Analyzing Selection Sort: 1st Loop

On first loop:

- Compare min element with each of 9 elements: cost = 9
- Do 1 swap: cost = 1

Total cost: 10

Analyzing Selection Sort: 2nd Loop

On second loop:

- Compare min element with each of 8 elements: cost = 8
- Do 1 swap: cost = 1

Total cost: 9

Analyzing Selection Sort: Last Loop

In the end, we have a list of size 1 left and don't have to do any work!

So work is 10 + 9 + 8 + … + 1 + 0

= 55

n

Now let's generalize to vectors of size *n*

First loop: n – 1 comparisons, 1 swap: cost = n

Second loop: cost = n – 1

…

Cost: n + (n – 1) + (n – 2) + … + 1 + 0

This is a famous sum!

Arithmetic Series

Memorize this!

$$\sum_{i=0}^{n} i = \frac{n(n+1)}{2}$$

That is,

$$0 + 1 + 2 + \cdots + n = \frac{n^2 + n}{2}.$$

How to Solve $\sum_{i=0}^{n} i$

Write the sum twice, once forwards and once backwards; then sum the two:

	0	+	1	+	…	+	n-1	+	n
+	n	+	n-1	+	…	+	1	+	0
=	n	+	n	+	…	+	n	+	n

How many n's are there in the sum? Answer: n+1.

Since we took twice the summation, we have to divide by 2,
Thus we have
n(n+1)/2.

Can also prove easily using induction, geometry, …

Visual Analysis

Counting based on code can be a pain;

Sometimes, a visual approach is simpler:

Original list	5	7	2	10	3	1	6	
Elements "touched" in first loop iteration	5	7	2	10	3	1	6	n
Elements "touched" in second loop iteration	1	7	2	10	3	5	6	n - 1
Elements "touched" in third loop iteration	1	2	7	10	3	5	6	n - 2
…								
Last iteration	1	2	3	5	6	7	10	1

Cost: O(1 + 2 + … + n)

ALGORITHMIC ANALYSIS

"Big O"

Big O notation:

O(n) measures *asymptotic complexity* of algorithm

Don't worry about the fancy language for now – this will be explained in CSCI 406!

What is important:

- In Big O, lower order terms and constants don't matter
- Only interested in how functions *grow* with size of n

Simplifying

Typically use the simplest term in expression:

- E.g., lower order polynomials can be ignored because they are completely *dominated* by higher order polynomials
  - O(n) not O(n + c)
  - O(n^2) not O(n^2 + n + c)
- Ignore constants
  - O(n) not O(3n)
  - O(n) not O(n/2)

Dominance relations (here a > b means a dominates b):

$n! > 3^n > 2^n > n^3 > n^2 > n \log n > n > \log n > 1$

Practice

Simplify the following:

O(n^3 + 4)

O(12n^2 – n + 1)

O(2^n + n^2)

O(n + n^2 + n log n)

Technicalities (for the curious)

- Defined:

  $f(n) = O(g(n))$ means $f(n) \le c\, g(n)$ for some c as $n \to \infty$

  More formally, $f(n) = O(g(n))$ if there exists c, n_0 such that $f(n) \le c\, g(n)$ for all $n > n_0$.

- The *asymptotic complexity* of *f* is upper bounded by *g*

Proof Sketch (for the curious)

(Just for show: not on any exams or homework!)

Prove: $3n^2 + n/4 + 1 = O(n^2)$

Find c, n_0 such that $(3n^2 + n/4 + 1) \le c\, n^2$ for all $n \ge n_0$

Choose c = 4, n_0 = 2

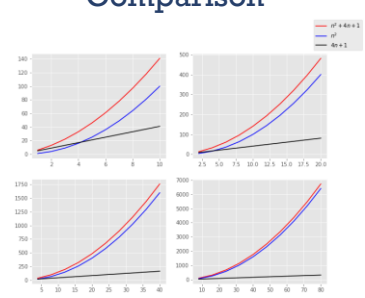
Inductive proof:

- Base case: $3(2^2) + 2/4 + 1 = 13½ \le 16 = 4(2^2)$
- Induction:
  - Suppose $3n^2 + n/4 + 1 \le 4\, n^2$
  - Show for n + 1:

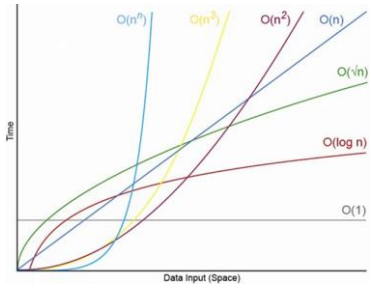
$3\,(n + 1)^2 + (n + 1)/4 + 1$	$\le$	$4\,(n + 1)^2$
$3\,(n^2 + 2n + 1) + n/4 + 1/4 + 1$	$\le$	$4\,(n^2 + 2n + 1)$
$3n^2 + 6n + 3 + n/4 + 1/4 + 1$	$\le$	$4n^2 + 8n + 4$
$(3n^2 + n/4 + 1) + 6n + 3¼$	$\le$	$(4n^2) + 8n + 4$

□

Asymptotic Complexity Comparison



Big-O Comparisons



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Why We Care 1

Comparison of different orders of functions as size of input n:

n	10	100	1000	10^6	10^9
log(n)	1	2	3	6	9
n	10	100	1000	1000000	1000000000
n log(n)	10	200	3000	6×10^6	9×10^9
n^2	100	10^4	10^6	10^{12}	10^{18}
2^n	1024	~10^{30}	~10^{300}	Forget it!	

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Why We Care 2

Assuming 2×10^{10} operations/second
(approximately the FP performance of a typical CPU c. 2011)*

n	10	50	100	10^6	10^9	10^{12}
log(n)	< 1 ns	< 1 ns	< 1 ns	1 ns	1 ns	2 ns
n	< 1 ns	< 1 ns	< 1 ns	50 µs	50 ms	50 s
n log(n)	< 1 ns	< 1 ns	1 ns	300 ms	450 ms	10 min
n^2	<1 ns	125 ns	500 ns	50 s	1.6 years	1.6 million years
2^n	50 ns	16 hours	1.5 trillion years			

*Why haven't I updated this slide? By (a misreading of) Moore's Law, CPUs could be about 4x as fast today as in 2011. How much should we care about this speedup?

Datasets of size 10^6 and above are commonplace!

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Up Next

- Start reading for Monday: Chapter 11
- Friday, February 1
  - Lab 4 – Analysis of Algorithms
  - APT 2 due
  - Project 2 - Mazes assigned
    - This project will be **interview graded** – we'll tell you more and give you a way to sign up for an interview soon!
- Monday, February 4
  - More about functions and the function call stack; review of recursion
  - Lab 4 due

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