CSCI 262
Data Structures

20 – Binary Search Trees

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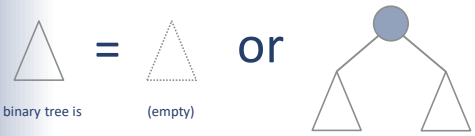
Review: Binary Trees

A binary tree is defined recursively:



A binary tree is (empty)

a *root node with* a *left* child and a *right* child, each of which is a binary tree.

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Review: Binary Tree Implementation

Just follow the recursive definition to get a simple implementation:

```
template <class T>
class binary_tree_node {
public:
   T data;
   binary_tree_node<T>* left;
   binary_tree_node<T>* right;
}
```

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Search Trees

Data structure for holding *comparable* elements

- Efficient searching, insertion, deletion
- Underlying structure for sets, maps (BSTs)
- Also used in database indexing (B-Trees)

The basic structure:

- Nodes hold *unique* data values and pointers to child nodes
- Data acts as a partitioning element in the tree:
  - Child nodes/trees to the left of the data element have value less than the data element
  - Child nodes/trees to the right have a value greater than the data element

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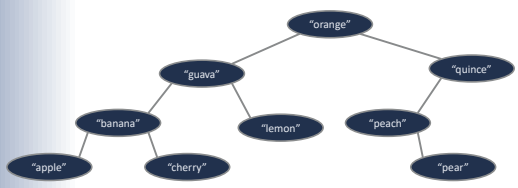
Binary Search Trees

Here's a binary search tree (BST):

- Nodes contain strings
- Left child subtree contains only values less than root
- Root value is less than all right child subtree values



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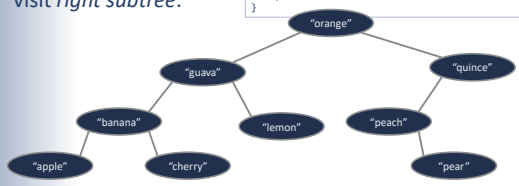
In Order Traversal

Visit *left subtree*,

visit *root*,

visit *right subtree*.

```
template <class T>
void print_inorder(binary_tree_node<T>* root) {
    if (root != NULL) {
        print_inorder(root->left);
        cout << root->data << " ";
        print_inorder(root->right);
    }
}
```



apple banana cherry guava lemon orange peach pear quince

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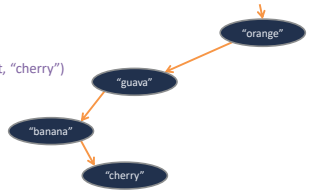
Searching

Example:
search (root, "cherry")




```
template <class T>
binary_tree_node<T>* search(binary_tree_node<T>* root, T val) {
   if (root == NULL) return NULL;
   if (val == root->data) return root;
   if (val < root->data) return search(root->left, val);
   else return search(root->right, val);
}
```

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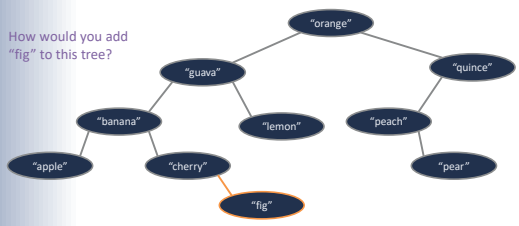
Inserting

Where do we insert an item into the tree?

How would you add "fig" to this tree?



Answer: put it where you expect to find it!

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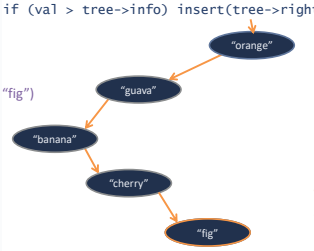
Inserting

```
template <class T>
void insert(binary_tree_node<T>*& root, T val) {
   if (root == NULL) root = new binary_tree_node<T>(val);
   else if (val < tree->info) insert(tree->left, val);
   else if (val > tree->info) insert(tree->right, val);
}
```

Example:
insert(root, "fig")



What if "fig" was already in our tree?

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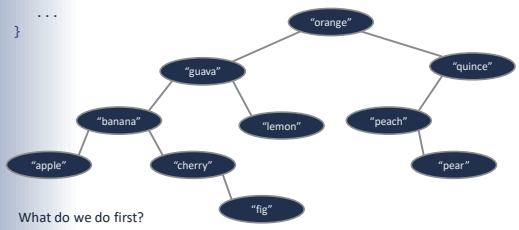
Removing

```
template <class T>
void remove(binary_tree_node<T>* root, T val) {
   // this is trickier!
   ...
}
```



What do we do first?
What is the base case?

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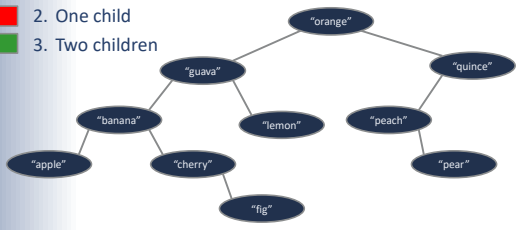
Removing

3 Cases when node is in tree:

1. No children
2. One child
3. Two children



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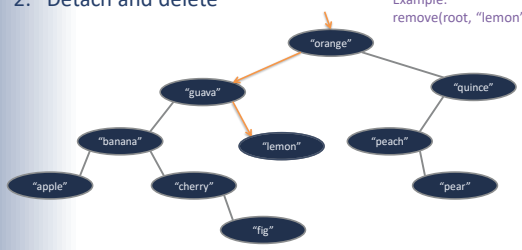
Removing Case 1: No Child

1. Find the item
2. Detach and delete

Example:
remove(root, "lemon")



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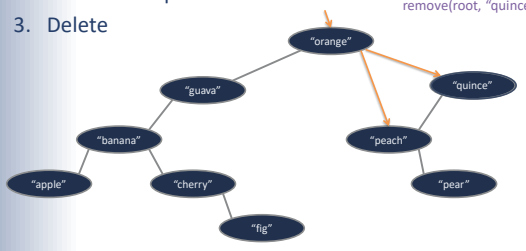
Removing Case 2: One Child

1. Find the item
2. Link child to parent
3. Delete

Example:
remove(root, "quince")



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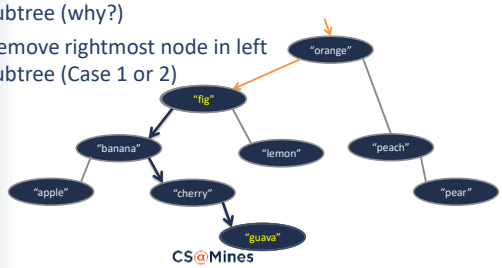
Removing Case 3: Two Children

1. Find the item
2. Swap with rightmost item in left subtree (why?)
3. Remove rightmost node in left subtree (Case 1 or 2)

Example:
remove(root, "guava")



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Removing: Code

```
template <class T>
void remove(binary_tree_node<T>*& root, T val) {
    if (root == NULL) return NULL;

    if (val < root->data) remove(root->left, val);
    else if (val > root->data) remove(root->right, val);

    else {  // item found!
        if (root->left == NULL || root->right == NULL) {
            binary_tree_node<T>* tmp;
            if (root->left == NULL) tmp = root->right;
            else tmp = root->left;
            delete root;
            root = tmp;
        }

        else {
            binary_tree_node<T>* tmp = root->left;  // find rightmost node
            binary_tree_node<T>* parent = root;     // in left subtree
            while (tmp->right != NULL) {
                parent = tmp;
                tmp = tmp->right;
            }
            root->data = tmp->data;        // copy data to root

            if (parent == root)            // detach and delete rightmost node
                root->left = tmp->left;    // in left subtree
            else
                parent->right = tmp->left;
            delete tmp;
        }
    }
}
```

Base case: item not found, do nothing.

Recursive calls to find item to delete.

Cases 1 and 2, both. Can you see why case 1 is handled by this block?

Case 3

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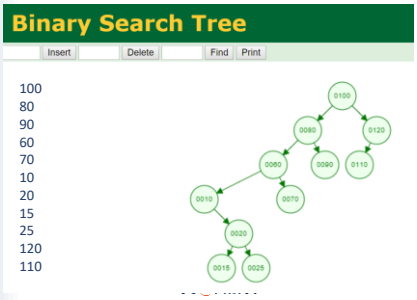
Practice With BSTs

https://www.cs.usfca.edu/~galles/visualization/BST.html



Analysis

What is the "big-O" complexity of:

- Searching?
- Inserting?
- Removing?

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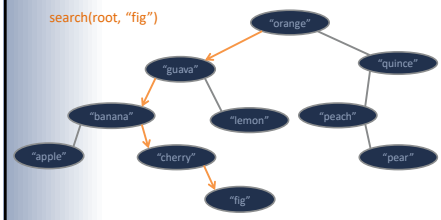
Complexity of Search

search(root, "fig")



How many recursive steps?

A. <= height of tree

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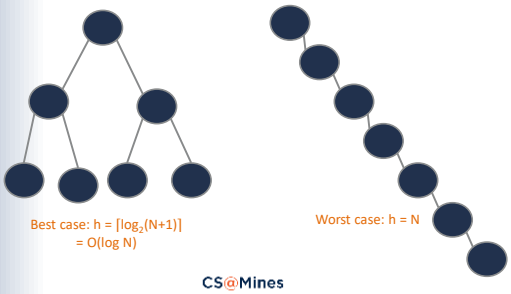
Height of Trees

So how high is a tree with N nodes?



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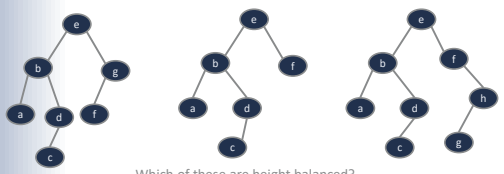
Height-Balanced Trees (AVL)

Again, a recursive definition:

- Left and right subtrees are height-balanced
- Left and right subtrees differ in height by no more than 1



Which of these are height balanced?

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Analysis on Balanced BSTs

When trees are balanced:

- Each subtree contains roughly half the nodes
- Each step down the tree roughly halves the problem

```
template <class T>
binary_tree_node<T>* search(binary_tree_node<T>* root, T val) {
   if (root == NULL) return NULL;
   if (val == root->data) return root;
   if (val < root->data) return search(root->left, val);
   return search(root->right, val);
}
```

Search, insert, delete are all O(log N)

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Self Balancing BSTs

- Trees become unbalanced through series of inserts and deletes
- Self-balancing: perform O(log N) or fewer operations to rebalance after insert, delete
- Examples of self-balancing BSTs:
  - Red-Black trees
  - AVL trees
  - Splay trees

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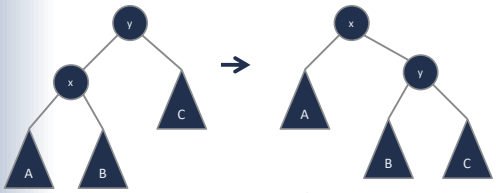
Rotations

We change the balance at a node via a *rotation*:



This is the *right rotation*. The *left rotation* is the mirror image of this one.

If the tree shown was a BST, is the new tree a BST?

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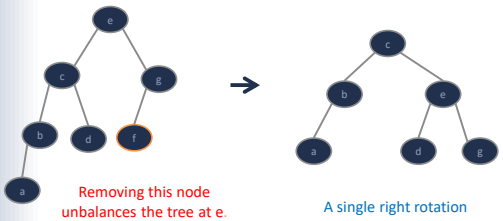
Rebalancing Example (AVL)



Removing this node unbalances the tree at e.

A single right rotation restores balance.

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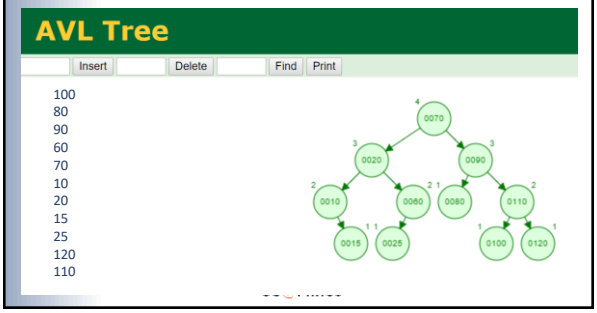
Tree Balancing (AVL)

https://www.cs.usfca.edu/~galles/visualization/AVLtree.html



Final Words

Why BSTs matter:

- Linux kernel: schedulers, ext3 filesystem, virtual memory, many more (Red-Black trees)
- Ordered set and map types (e.g., C++ STL, Java) (Red-Black trees again!)
- Database indexing (B-trees – not exactly BSTs, but related)
- Filesystem metadata indexing (B-trees or R-B)
- Lurking in your favorite OS?

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Up Next

- Reading for Wednesday: Chapter 10
- Wednesday, April 24
  - Inheritance
- Friday, April 26
  - Lab 12 – Inheritance
  - Extra Credit APT due

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